5. The coefficient $a_{12}$ = 0.5 tells that the NOx increases by an additional half ppm when both oxygen and air temperature increase or decrease together. That is, some synergy/moderation exists. If oxygen and air preheat differ in sign (high O2, low air preheat, or vice versa) then the interaction lowers NOx by 0.5 ppm more than when considering $x_1$ and $x_2$ alone. One may capture this effect only if the experimental design varies both factors simultaneously as factorial designs do. These interaction effects account for the curved contour lines and the uneven spacing in Figure 12.6-1. Visually these interactions appear to be slight; mathematically, the largest interaction is only about 40% of the smallest first-order effect.
6. This examination shows that NOx increases with an increase in air temperature, furnace temperature, and oxygen concentration. Positive interactions between the temperatures and oxygen concentration add to these effects.

## 12.7 USE OF THE NORMAL PROBABILITY PLOT TO ASSESS SIGNIFICANCE

The experimental design of Equation 12.5-1 does not have any term for experimental error, and without an estimate, one cannot decide if a coefficient represents random noise or is truly significant. Genuine replicates discussed in the next chapter are the best tool for estimating experimental error. Notwithstanding, higher-order interactions are often negligible and may represent no real effect at all. In such a case, they provide an estimate of error. In the absence of genuine replicates, the distribution of the coefficients themselves provides an estimate. If the terms are merely taking up random error but no more, they will tend to fall in a straight line on a normal probability plot. Significant effects should deviate from a straight line. One may create the normal probability plot with the following procedure.¶

1. Order the coefficients from lowest to highest to form the ordinate values for the plot excluding $a_0$.
2. Number each of the $m$ coefficients, from smallest to greatest, as $k = 1 \ldots m$.
3. Transform the numbers according to $z_k = (k - 0.5)/m$ to give the percentiles.
4. Calculate the ordinates of the normal probability curve using the inverse cumulative probability distribution of $z_k$.**
5. Plot the ordered coefficients against these values.
6. Construct a line by plotting the normal ordinate multiplied by the standard deviation of the coefficients against the normal ordinate. This is the line s = 1.

Alternatively, one may use Lenth's method[3] to calculate a pseudo-standard error (PSE), using a multiple of the median average deviation via the Real-Statistics command, =MAD(**a**)*1.4826, where **a** is the coefficient vector. The 1.4826 is derived from the formula that the standard deviation is approximately 1.4826 the median average deviation. Dividing each coefficient by the PSE gives an approximate t-value that may be used to assess the significance of each effect.

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¶ Adapted from Nelson, P.R., Design and analysis of experiments, in Handbook of Statistical Methods for Scientists and Engineers, Wadsworth, H.M., Ed., McGraw-Hill, New York, 1989, chap. 14.

** The Excel function =NORMSINV($z_k$) will do this.

## REFERENCES

1. Myers, R.H. and Montgomery, D.C., Chapter 3, Two-level factorial designs, in *Response Surface Methodology, Process and Product Optimization Using Designed Experiments*, 2nd ed., John Wiley & Sons, New York, 2002, p. 85.
2. Box, G.E.P. and Draper, N.R., Chapter 14, Variance-optimal designs, in *Empirical Model-Building and Response Surfaces*, John Wiley & Sons, New York, 1987, p. 484.
3. Lenth, R. V. (1989). Quick and Easy Analysis of Unreplicated Factorials. *Technometrics*, 31(4), 469–473.