

6. This gives the same $O_2 \times O_2$ leverage plot as the permutation method of Example 16.7-1.
- Equation 16.8-3 gives the leverage points (Cells E3:F30). Regressing the vector held in Cells F3:F30 against the matrix held in D3:E30 gives the coefficients (Cells L37:L38) for the line of fit.
 - Incrementing 21 values (or whatever is necessary for a smooth curve) between the maximum and minimum leverage values in Cells C35:C55 provides the abscissa for the line of fit.
 - Multiplying the matrix of Cells B35:C35 by the regressed constants expresses the ordinate held in Cells D35:D55.
 - Cells E35:J55 hold the confidence and prediction intervals and limits derived in the same way as the previous example.
 - The plot located in the space of Cells M21:T44 graphs the results and is identical to the $O_2 \times O_2$ graph of Example 16.7-1.
 - Thus, one may define the specification vector to produce any desired leverage plot, such plots being identical to those given by the permutation matrix of Example 16.7-1.

16.9 CONFIDENCE LIMITS, PREDICTION LIMITS, AND PROFILE PLOTS FOR \hat{y} .

Note that the same methodology may be used to construct confidence and prediction intervals for the actual vs. predicted plot of y vs. \hat{y} . In such a case, $y = \hat{y} + \varepsilon$. Then $\mathbf{Y}^T = (\mathbf{1} \ \hat{\mathbf{y}})$, and for a particular value, $\hat{\mathbf{y}}^T = (1 \ \hat{y})$. With these substitutions, Equations 16.7-5 through 16.7-7 yield Equations 16.9-1 through 16.9-3.

$$\mathbf{CI}_{\hat{y}} = t_{crit} S \sqrt{\hat{\mathbf{y}}^T (\mathbf{Y}^T \mathbf{Y})^{-1} \hat{\mathbf{y}}} \quad \mathbf{PI}_{\hat{y}} = t_{crit} S \sqrt{1 + \hat{\mathbf{y}}^T (\mathbf{Y}^T \mathbf{Y})^{-1} \hat{\mathbf{y}}} \quad (16.9-1a, b)$$

$$\mathbf{LCL}_{\hat{y}} = \hat{y} - \mathbf{CI}_j \leq \hat{y} \leq \hat{y} + \mathbf{CI}_j = \mathbf{UCL}_{\hat{y}} \quad (16.9-2)$$

$$\mathbf{LPL}_{\hat{y}} = \hat{y} - \mathbf{PI}_j \leq \hat{y} \leq \hat{y} + \mathbf{PI}_j = \mathbf{UPL}_{\hat{y}} \quad (16.9-3)$$

In such a case, one may increment $\hat{y}_{\min} \leq \hat{y} \leq \hat{y}_{\max}$ to plot $\mathbf{LCL}_{\hat{y}} \leq \hat{y} \leq \mathbf{UCL}_{\hat{y}}$ and $\mathbf{LPL}_{\hat{y}} \leq \hat{y} \leq \mathbf{UPL}_{\hat{y}}$, giving the actual-vs.-predicted plot shown in Figure 16.7-4, lower-left corner. Note that these differ from the CI and PI calculated from \mathbf{X} , which are multivalued in \mathbf{y} . If one desires a least-squares fit to the multivalued intervals, one may use $\widehat{\mathbf{CI}} = \left(\frac{\sum \mathbf{CI} \cdot \mathbf{CI}_{\hat{y}}}{\sum \mathbf{CI}_{\hat{y}}^2} \right) \mathbf{CI}_{\hat{y}} \approx \mathbf{CI}$ and $\widehat{\mathbf{PI}} = \left(\frac{\sum \mathbf{PI} \cdot \mathbf{PI}_{\hat{y}}}{\sum \mathbf{PI}_{\hat{y}}^2} \right) \mathbf{PI}_{\hat{y}} \approx \mathbf{PI}$ which provide wider limits for the least-squares fits of the multivalued data, typically $\frac{\sum \mathbf{PI} \cdot \mathbf{PI}_{\hat{y}}}{\sum \mathbf{PI}_{\hat{y}}^2} \approx 1.1$ and $\frac{\sum \mathbf{CI} \cdot \mathbf{CI}_{\hat{y}}}{\sum \mathbf{CI}_{\hat{y}}^2} \approx 2$.

Perhaps the simplest representation of a multidimensional surface is to take a two-dimensional slice (profile) along every factor axis and present them together. Figure 16.9-1 shows a triplet of profile plots for burner stability as a function of fuel pressure, furnace load, and airflow for a particular burner.