6. This gives the same O2 × O2 leverage plot as the permutation method of Example 16.7-1.
   a. Equation 16.8-3 gives the leverage points (Cells E3:F30). Regressing the vector held in Cells F3:F30 against the matrix held in D3:E30 gives the coefficients (Cells L37:L38) for the line of fit.
   b. Incrementing 21 values (or whatever is necessary for a smooth curve) between the maximum and minimum leverage values in Cells C35:C55 provides the abscissa for the line of fit.
   c. Multiplying the matrix of Cells B35:C35 by the regressed constants expresses the ordinate held in Cells D35:D55.
   d. Cells E35:J55 hold the confidence and prediction intervals and limits derived in the same way as the previous example.
   e. The plot located in the space of Cells M21:T44 graphs the results, and is identical to the O2 × O2 graph of Example 16.7-1.
   f. Thus, one may define the specification vector to produce any desired leverage plot, such plots being identical to those given by the permu-tation matrix of Example 16.7-1.

16.9 CONFIDENCE LIMITS, PREDICTION LIMITS, AND PROFILE PLOTS FOR $\hat{\mathbf{y}}$.

Note that the same methodology may be used to construct approximate confidence and prediction intervals for the actual vs. predicted plot of y vs. ŷ. In such a case, $y = \hat{y} + \varepsilon$. Then $\hat{\mathbf{Y}}^{\mathrm{T}} = (\mathbf{1} \quad \hat{\mathbf{y}})$, and for a particular value, $\hat{\mathbf{y}}^{\mathrm{T}} = (1 \quad \hat{y})$. With these substitutions, Equations 16.7-5 through 16.7-7 yield Equations 16.9-1 through 16.9-3.

$$\mathbf{CI}_{\hat{\mathbf{y}}} = t_{crit} s\sqrt{\hat{\mathbf{y}}^{\mathrm{T}}(\mathbf{Y}^{\mathrm{T}}\mathbf{Y})^{-1}\hat{\mathbf{y}}} \qquad \mathbf{PI}_{\hat{\mathbf{y}}} = t_{crit} s\sqrt{1 + \hat{\mathbf{y}}^{\mathrm{T}}(\mathbf{Y}^{\mathrm{T}}\mathbf{Y})^{-1}\hat{\mathbf{y}}} \tag{16.9-1a, b}$$

$$\mathbf{LCL}_{\hat{\mathbf{y}}} = \hat{\mathbf{y}} - \mathbf{CI}_j \leq \quad \hat{\mathbf{y}} \quad \leq \hat{\mathbf{y}} + \mathbf{CI}_j = \mathbf{UCL}_{\hat{\mathbf{y}}} \tag{16.9-2}$$

$$\mathbf{LPL}_{\hat{\mathbf{y}}} = \hat{\mathbf{y}} - \mathbf{PI}_j \leq \quad \hat{\mathbf{y}} \quad \leq \hat{\mathbf{y}} + \mathbf{PI}_j = \mathbf{UPL}_{\hat{\mathbf{y}}} \tag{16.9-3}$$

In such a case, one may increment $\hat{y}_{\min} \leq \hat{y} \leq \hat{y}_{\max}$ to plot $\mathrm{LCL}_{\hat{\mathrm{y}}} \leq \hat{y} \leq \mathrm{UCL}_{\hat{\mathrm{y}}}$ and $\mathrm{LPL}_{\hat{\mathrm{y}}} \leq \hat{y} \leq \mathrm{UPL}_{\hat{\mathrm{y}}}$, giving the actual-vs.-predicted plot shown in Figure 16.7-4, lower-left corner. These are merely approximations as the true CI and PI are functions of **X**, not **y**. Notwithstanding, they proved useful visual representations. The least-squares corrections $\widehat{\mathbf{CI}} = \left(\frac{\sum CI \cdot CI_{\hat{y}}}{\sum CI_{\hat{y}}^2}\right)\mathbf{CI}_{\hat{\mathbf{y}}} \approx \mathbf{CI}$ and $\widehat{\mathbf{PI}} = \left(\frac{\sum PI \cdot PI_{\hat{y}}}{\sum PI_{\hat{y}}^2}\right)\mathbf{PI}_{\hat{\mathbf{y}}} \approx \mathbf{PI}$ provide further improvements.

Perhaps the simplest representation of a multidimensional surface is to take a two-dimensional slice (profile) along every factor axis and present them together. Figure 16.9-1 shows a triplet of profile plots for burner stability as a function of fuel pressure, furnace load, and airflow for a particular burner.